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PORTFOLIO OPTIMIZATION ON CROATIAN CAPITAL MARKET

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Abstract:
Purpose of this paper was to research portfolio optimization problem on Croatian capital market using Markowitz theory. Research systematically investigated the selection of securities, and defined the importance of using fundamental analysis when selecting the best combination of securities. Since fundamental analysis involves a large number of indicators, this paper selected key indicators that enable a complete and quick securities review on the market. This paper clarifies diversification effect and influence of the correlation coefficient on diversification. Two basic types of assets (stocks and cash funds) have been chosen to build the optimal portfolio. Cash funds were selected because they represent a form of risk-free investment, while stocks were chosen because of the high level of return which they achieve. At the end of paper, optimal portfolio was calculated with an excellent yield of 1.82% and deviation of 5.77% on a monthly basis which corresponds to the minimum deviation of the selected stocks. Calculated optimal portfolio achieves better expected value than investing in stock index CROBEX, which for the same period achieves the expected result of -0.02%.

Keywords: optimal portfolio, diversification, asset allocation, stock, cash fund, risk, fundamental analysis.

Jel Classification: G11, C61

INTRODUCTION

In this paper Markowitz theory was used (1952; 1959) for construction of an optimal portfolio on Croatian capital market. One of the study aims was to explain, how to achieve maximum profits with limited portfolio risk by modifying the shares of individual assets.

Portfolio is the amount of stocks, bonds or other financial or real assets, owned by individual, group of people or a company with the aim of making profits. Any investor who thinks rationally will choose the portfolio from the efficient frontier because it promises the best combination of the risk and return. Efficient frontier represents the

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ultimate limit, which is bounded by the set of all securities in the portfolio. Part of the boundary which is located above the point of minimum variance is called efficient, while part of the border that is located below the point of minimum variance is called ineffective. The optimal portfolio of an individual investor will be one that meets the interests of individual investors with respect to return and risk. When it's talking about the risk it is important to clarify two basic concepts that enjoy risk–hazard and speculation. The risk premium presents the factor that separates hazard from speculation. Economically, the hazard is a prerequisite to enjoy the risk, while speculation presupposes an adequate risk premium to compensate the risk that is accepted.

One of the most important decisions that an investor has to make is a decision on the assets allocation. Asset allocation is the distribution of assets in the categories of assets such as: money market instruments, fixed income securities, equities, real estate, precious metals, certificates, funds, etc. Once the investor determines what type of assets he wants in his portfolio, he precisely selects individual securities to buy. Most investors believe that asset allocation is the most important decision in building the portfolio.

Another important decision that investors make when choosing stocks is in the amount of risk they are willing to submit. The amount of risk that the investors are willing to bear depends on their own preferences, age, health, job expectations, etc. — these are just some factors that affect the amount of risk they are willing to bear. When it's talking about risk exposure it is important to mention two basic types of risk; specific and systemic securities risk. Systemic risk is caused by macro events, it is the risk that affects the entire market. According to Bogdan, Baresa and Ivanovic (2010) systemic risk includes market risk, or the risk that comes from macroeconomics and can't be eliminated by diversification.

Systemic risk is closely linked to capital market conditions, factors that are related to environment in which securities are determining it (for example: events such as changes in market interest rates, general recession, natural disasters, wars, political events, economic growth etc. affect the entire market regardless to the sector the securities belong). Specific risk is caused by micro factors on enterprise level. Investing in a large number of securities reduces specific risk — therefore it is possible to diversify it.

Summarized; the key elements involved in the decision making process are: resources, or the amount of money with which the investor is willing to invest, the estimated risks and returns as well as liquidity. Liquidity is an important condition by which an investor can easily buy or sell a security. Liquidity presents a speed and ease by which an asset can be sold. It must be taken into consideration that liquidity is a limiting factor in asset allocation on the Croatian capital market. Bogdan, Baresa and Ivanovic (2012) examined the liquidity of the shares in the Croatian capital market and as key factors pursuant to which the investors are more inclined to invest are size of market capitalization and number of issued shares.

This paper presents the criteria that were used in selecting securities for the construction of an optimal portfolio, assuming that the funds were allocated to stocks and a cash funds. One of the aims of this paper is to present way of constructing an optimal portfolio by calculating shares of individual asset in a way that maximizes utility considering the expected returns, risk and correlations of assets and risk tolerance.

1. BUILDING A PORTFOLIO

A rational investor will always try to maximize returns relative to the risk that is submitted, therefore, the construction of the portfolio can be displayed through next five steps.

The first step in building a portfolio lies in the fact that the investor determines the length of time horizon and the amount of risk it can tolerate. Regarding the time horizon, it raises a question whether the investor is planning a long term or short term investment?

The second step consists in selecting the appropriate policy. When it's deciding about choosing the appropriate policy it is very important to determine the limits of the minimum and maximum amount of money that will be allocated to investment. Investors must decide for a specific combination of securities they wish to have in the portfolio.

The third step would be the determination and application of portfolio strategy. Portfolio strategies can be active and passive. Selecting active portfolio strategy investor constantly evaluates and makes decisions to build a portfolio with higher earnings, and thus beat the market average. Active investor always tries to be faster than the information that the market will reflect. As for the cost of investment, the active investor is exposed to higher costs for frequent transactions; therefore, profits must be greater than the costs of transactions and the market average in the sum. Assumption that binds to the active strategy is that the market is not efficient, and it is possible to find favorable opportunities for investment.

Passive strategy implies buying securities with the aim of risk diversification, and keeping them in the long therm. Usually these securities are part of the stock, bond, index, etc. Passive portfolio management considers that the prices of the securities are very close to their „fair" value, so investor should instead of trying to „outmatch" the market, passive investor selects portfolio depending on his risk tolerance and considers that he will be fairly rewarded. Investing in index such as e.g.: DAX, Standard and Poor's 500, CAC, etc. is considered investing in a widely diversified portfolio, which makes a good example of passive investment. Cristiana Tudor (2012) in research shows an example where the passive strategy is inferior then active strategy.

After the aims, policies and strategies are determined, the fourth step is the selection of assets. The investor selects the asset in a way that using analytics chooses assets that will at certain level of risk give the maximum return. One question that arises is: ″How many securities should be included in the portfolio?″ The simple answer to this question doesn't exist. This question depends on a variety of accompanying factors such as are: the investment time horizon, the investor ability to successfully predict price movements, diversification which allows investor to reduce specific risk etc.

The fifth step assumed constant portfolio revision. After investor sets goals, policies, apply chosen portfolio strategy, and analyze securities, often the information will be outdated, so it is needed a constant portfolio revision and reconsideration of own decisions.

2. ASSET ALLOCATION

Considering that the situation in the Croatian capital market in the last three years is very bad (when it's talking about liquidity of securities) initial requirement was to choose liquid securities on a monthly basis, in order to calculate returns.

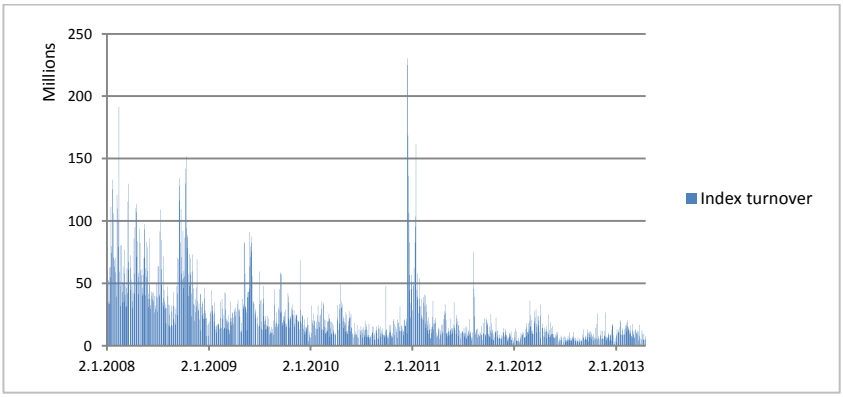

Figure 1. The Zagreb Stock Exchange equity index turnover in millions HRK

Chart above shows movement of Zagreb Stock Exchange equity index turnover in the last five years. Based on the chart it can be clearly seen drop of stock turnover on ZSE.

In this paper 70 securities which were traded each month for a period of three years from January 1st 2010 to December 31st 2012 were included in research. Since stocks represent risky form of investment, as a counterweight there were also cash funds chosen because they represented form of low risk investment. For research purposes, cash funds were analyzed, total of 9 among which we also made further selection.

In the first step historical monthly returns were calculated, based on the last price in the trading month. On the official website of the Zagreb Stock Exchange returns are calculated in the same way. Simple calculation returns were calculated with no dividend yield, according to the formula below.

$$R_{im} = \frac{P_{im} - P_{im-1}}{P_{im-1}} \quad (1)$$

R_{im} = Return of stock i in month m
P_{im} = Price of stock i in month m

The aim was to select a set of stocks which will be on the shortlist for constructing the optimal portfolio. The first elimination criterion of selection was achieved positive average return. Selection excluded 42 stocks that had achieved negative average returns in a period of three years. The remaining 28 stocks were reduced to 9 stocks which had satisfying fundamental results and the average return above average return of cash funds — that was second elimination criterion. Since cash funds are considered risk–free investment, if some stocks had equal or lower expected return than cash funds, priority is given to cash funds due to lower risk. Third reason for the elimination of

individual stocks was too high correlation with other stocks, which would prevent efficient diversification. Therefore, stocks with a high coefficient of correlation and weaker fundamentals were removed. Important criteria were also: enough big market capitalization, and business transparency. For simplicity of data presentation below are shown stocks that have been selected to build the optimal portfolio.

Table 1. Financial indicators of selected stocks

	M. Cap (HRK - 000)	*P/E*	*P/B*	*P/S*	*P/EBITDA*	*P/EBIT*	*EV/EBITDA*	*EV/EBIT*	*ROA(%)*	*ROE(%)*
INDUSTRY										
KOEI-R-A	1,517.550	9.78	0.90	0.66	9.82	17.50	14.25	25.39	4.65	9.17
ERNT-R-A	1,551.372	17.88	2.38	0.90	16.57	28.96	12.75	22.28	6.83	13.31
CONSTRUCTION SECTOR										
VDKT-R-A	147.093	5.14	0.75	0.17	1.73	3.48	3.33	6.71	4.05	14.49
TOURISM										
LRH-R-A	514.489	N/A	0.59	2.62	12.36	53.33	12.05	52.00	0.80	0.90
RIVP-R-A	1,340.352	20.90	0.71	1.39	4.64	13.17	6.64	18.85	2.16	3.41
KORF-R-A	783.312	11.53	0.50	0.74	2.38	5.73	4.12	9.94	2.15	4.32
MAIS-R-A	547.216	45.08	0.54	0.90	3.22	11.14	3.23	11.18	0.59	1.19
NUTRITIONAL SECTOR										
KRAS-R-A	545.355	N/A	0.81	0.49	6.20	18.26	9.14	26.92	0.54	1.07
LEDO-R-A	1,287.994	12.29	1.67	1.40	10.42	13.01	10.57	13.19	9.43	13.58

Based on the data from the previous table it can be noticed that the selected stocks for the analysis are from industrial, construction, tourism and nutritional sector. Diversification effect is enhanced because the asset is allocated in several sectors. Balli and Balli (2011) claim that diversification within those Euro-wide basic industry sectors (basic resources, food and beverage, healthcare, oil and gas, retail services, and utility) might also be more efficient in reducing the portfolio risk. When we create a portfolio with these sector equity indices only, we show that investors will gain a higher return with less risk compared to creating a portfolio with entire Euro sector equity indices or with Euro national equity indices only.

Following text explains financial indicators, which have played important role in the allocation of asset.

Market capitalization is an indication of volume and turnover of selected traded company. It is calculates by multiplying the issued stocks and market price. According to Marc Reinganum (1999) companies with smaller market capitalization over a long investment horizon generate higher returns, but they are also exposed to higher risk. Marc Reinganum also describes a market capitalization as one of the most important determinants of portfolio returns. In creating an optimal portfolio there was set a minimum of market capitalization in an asset allocation at level of 500,000.000 HRK.

P/E relationship is considered to be one of the key indicators for financial analysts. *P/E* ratio is the market price–earnings stocks. Stocks that have low *P/E* ratio are considered better opportunities than stocks with high *P/E* ratio. For example, in the previous table stock VDKT-R-A has the lowest *P/E* = 5.14 so it can be explained like investor pays 5.14 HRK for 1 HRK of current profits. According to Park (2000) a high *P/E* ratio may not necessarily mean that the stocks are overvalued. Investors certainly should not use a high *P/E* as a standalone alarm to signal a sale.

P/BV is calculated by dividing the market price of the stock by the book value. Investors believe that stocks with low ratios of *P/B* are good opportunities for shopping, but that opinion can't be taken independently. From the previous table it can be seen that the most stocks are *P/B*<1, according that it can be concluded that it is word mainly about undervalued stocks, except stocks ERNT-R-A which has *P/B* = 2.38.

P/S presents the ratio that divides current market stock price and the value of stock income. Value of stock income is determined by dividing the total income of the firm and the total number of issued stocks. Investors prefer the smaller ratio. *P/S* ratio is usually not taking independent for stock analysis, it is usually used in conjunction with other indicators. Most suitable indicator *P/S* have stocks VDKT 0.17, KRAS 0.49 and KOEI 0.66.

P/EBITDA and *P/EBIT* are indicators that have similar concept as an indicator of *P/E*. *EBITDA* represents operating profit, which excludes interest, taxes, depreciation and time adjustment. *EBIT* represents operating profit. Of all stocks, best stock is VDKT which achieves *P/EBITDA* 1.73 and *P/EBIT* 3.48.

EV/EBITDA and *EV/EBIT* are indicators which present ratio of company value and earnings that exclude certain items. Company value is calculated as follows:

$$EV = capital\ market\ value + market\ debt\ value - money\ and\ equivalents \quad (2)$$

Enterprise value is a better indicator of the market capitalization for acquisition; because it takes into account the debt owed by the acquirer takes over. *EBITDA* and *EBIT* are defined in previous indicator. Indicator of low *EV/EBITDA* and *EV/EBIT* may mean there is underestimated stock company. Indicators of low *EV/EBITDA* and *EV/EBIT* generate following stocks VDKT, KORF and MAIS.

ROA presents the coefficient of profitability and return on total assets. It calculates by dividing the net profit and total assets. According Ivanovic (1997) this coefficient is quite inappropriate, because it is calculated from net income as the size taken after interest paid to creditors, and because the creditors have the right to claim the property of company until its settlement, it is much more realistic the coefficient, which expresses the relationship between gross operating income and total assets. Investors prefer companies with higher returns on assets. Greatest return on assets generate stocks LEDO 9.43% ERNT 6.83%, and KOEI 4.65%.

ROE is an indicator that presents return on equity. It is obtained by dividing the net profit and equity. Investors prefer higher *ROE* indicator. Stocks that generate higher *ROE* indicator are VDKT 14.49 LEDO 13.58 and ERNT 13.31.

Once, that the companies are selected it was necessary to select the cash funds. Based on the following table, selection has been made for two cash funds, under condition that one fund has the majority of assets contained in HRK, and another fund in EUR. The next condition funds had to meet was they had to be present on the market for at least 3 years. In order to partially disperse the currency risk two funds were chosen, they were denominated in HRK and in EUR.

Table 2. Overview of cash funds

(%)

Name of the Cash Fund	Return			$\bar{x}$	VaR	The currency exposure of the Fund	
	2010	2011	2012			EUR	HRK
LOCUSTA CASH	4.01	3.89	4.60	0.34	0.10	5.00	95.00
AGRAM EURO CASH	4.08	3.09	3.53	0.29	0.13	96.45	3.55
ERSTE EURO MONEY	3.20	2.85	4.03	0.28	0.17	99.35	0.65
PBZ EURO NOVČANI	3.53	2.50	3.79	0.27	0.10	99.89	0.11
ALLIANZ CASH	3.43	2.69	3.48	0.26	0.06	2.56	97.44
ERSTE MONEY	3.07	2.40	3.65	0.25	0.10	0.02	99.98
VB CASH	3.22	1.57	3.35	0.22	0.48	4.16	95.84
PBZ NOVČANI FOND	3.05	2.12	2.87	0.22	0.09	0.00	100.00
ZB EUROPLUS	2.51	1.99	2.51	0.19	0.08	96.60	3.40

$\bar{x}$ represents the monthly average returns for the period of 3 years 2012, 2011 and 2010. Based on the data from the previous table, LOCUSTA CASH, had the best results achieved of all available funds, the average monthly return 0.34% made him the best choice in the case of cash funds, he had most of the resources contained in the HRK. Considering the fund which has highest historical returns, and most of money contained in EUR that was AGRAM EURO CASH with an average monthly return 0.29%.

4. DATA AND RESEARCH

The assumption under which this work was written is that historical returns are good future predictors. With this assumption, the expected return, standard deviation and variance are measured using historical data. Numerous articles are written, about stock return predicting, but predictability is not yet discovered. Some of the academic papers that have dealt with these issues Ferreira and Santa-Clara (2011) said that there are better methods of forecasting from historical data or regression, they proposed forecasting separately the dividend-price ratio, the earnings growth, and the price-earnings growth components of stock market returns: the sum-of-the-parts method. Campbell and Yogo (2006) tested indicators of price-earnings ratio and the dividend price ration, they have proven that there is predictable component in forecasting stock returns only it is difficult to detect without careful use of statistical tests.

When it's talking about the risk, it differs risk we can diversify and the risk that can't be diversified. If an investor invests all his money in one stock company, that investment depends on the specific risks of the stock company. However, if he expands investment in other assets, he will diversify the specific risk. This work used risk control tool–diversification.

Since all stocks are ordinary, in the continuation of paper stock tickers will be shown without indication R-A. Considering money funds, only first word from the name will be used.

Table 3. Calculated expected returns, variances and deviations of the stocks and funds

	ERNT	KOEI	KORF	KRAS	LEDO	LRH	MAIS	RIVP	VDKT	AGRAM	LOCUSTA
E(r)	0.42%	1.34%	3.56%	0.55%	0.65%	1.10%	0.49%	1.51%	1.46%	0.29%	0.34%
σ^2	63.22	39.69	183.77	33.28	59.71	127.87	148.70	81.10	266.39	0.00	0.02
σ	7.95	6.30	13.56	5.77	7.73	11.31	12.19	9.01	16.32	0.05	0.14
E(r)/ σ	0.05	0.21	0.26	0.10	0.08	0.10	0.04	0.17	0.09	5.52	2.46

Based on the table 3, it can be noticed that all assets have positive expected returns. Stock KORF has the highest return, but also the highest standard deviation, while cash funds had lower expected returns, but almost negligible standard deviation. Last row of the table 3 indicates *Sharpe ratio* known as *Sharpe index*, or as a reward for the variability which measures the ratio of return and risk. It applies as a measure of return per unit of risk. The risk is presented by standard deviation. It is preferable that the *Sharpe index* is higher as can be. According to Sharpe (1994) despite measured by ex-ante or ex post, it is essential that *Sharpe index* is measured using the arithmetic mean and standard deviation of the differential return. Bohdalova and Gregus (2012) created the portfolio that produces a portfolio returns distribution that has the best performance metrics, the highest Sharpe ratio. They have used empirical returns joint distributions, they were not limited on the multivariate normality assumption of the standard mean-variance analysis. They have shown how copulas provide a very flexible tool for modelling this joint distribution.

The standard deviation of the portfolio is the weighted average of the standard deviation of the securities in the portfolio only when the correlation is perfectly positive. In the case of a perfect correlation between stocks, diversification would not be effective. Therefore, it can be concluded whenever $\rho < 1$ standard deviation of portfolio is less than weighted average of the standard deviation of the securities in the portfolio. In order to demonstrate the effect of diversification on the standard deviation in the table 4 is showed an example of a portfolio that contains 2 stocks with the highest standard deviation (VDKT and KORF), then a portfolio with 3 stocks (VDKT, KORF and MAIS), and portfolio with 4 stocks (VDKT, KORF, MAIS and LRH). From the table 3 it is possible to read the individual deviations of securities and compare to portfolio deviations in the table 4.

Table 4. The diversification effect

2 asset portfolio	w_{KORF}	w_{VDKT}	Σw_i	σ_p		
	0.50	0.50	1.00	12.03		
3 asset portfolio	w_{KORF}	w_{VDKT}	w_{MAIS}	Σw_i	σ_p	
	0.33	0.33	0.33	1.00	10.43	
4 asset portfolio	w_{KORF}	w_{VDKT}	w_{MAIS}	w_{LRH}	Σw_i	σ_p
	0.25	0.25	0.25	0.25	1.00	8.54

Based on the table above, the effect of diversification which aims to remove specific risk can be confirmed. The portfolio which consists of two assets has σ_p = 12.03, the portfolio which consists of three assets has σ_p= 10.43, and portfolio that consists of four assets has σ_p= 8.54. In order to prove for the rest of the asset that the diversification is worth, in continuation is showed correlation matrix a chosen asset.

Table 5. Correlation matrix

	ERNT	KOEI	KORF	KRAS	LEDO	LRH	MAIS	RIVP	VDKT	AGRAM	LOCUSTA
ERNT	1.00										
KOEI	0.26	1.00									
KORF	0.14	0.52	1.00								
KRAS	0.03	-0.11	0.06	1.00							
LEDO	0.53	0.55	0.39	0.06	1.00						
LRH	-0.08	-0.10	0.08	0.07	-0.03	1.00					
MAIS	0.13	0.32	0.34	-0.14	0.26	0.06	1.00				
RIVP	0.21	0.11	0.37	0.13	0.10	-0.09	0.07	1.00			
VDKT	0.37	0.41	0.29	0.02	0.39	0.05	0.35	0.26	1.00		
AGRAM	0.25	0.32	-0.05	-0.16	0.22	0.29	0.21	-0.11	0.30	1.00	
LOCUSTA	-0.02	-0.06	-0.01	0.07	-0.20	0.44	-0.05	-0.08	-0.20	0.45	1.00

Based on the correlation matrix above, it can be seen that the stocks are weakly correlated with each other, except LEDO stock that achieves correlation coefficient with ERNT 0.53 while KOEI achieves correlation 0.52 with KORF stock. Cash funds AGRAM and LOCUSTA have mutual correlation coefficient 0.45 but with stocks they generate very poor correlation.

For the purpose of calculating the variance of the portfolio in continuation is presented covariance matrix from which will values retrieve.

Table 6. Covariance matrix

	ERNT	KOEI	KORF	KRAS	LEDO	LRH	MAIS	RIVP	VDKT	AGRAM	LOCUSTA
ERNT	63.22										
KOEI	13.08	39.69									
KORF	15.22	44.35	183.77								
KRAS	1.18	-3.85	4.72	33.28							
LEDO	32.84	26.58	40.81	2.78	59.71						
LRH	-6.81	-7.02	11.82	4.50	-2.50	127.87					
MAIS	12.43	24.60	56.00	-9.69	24.86	8.26	148.70				
RIVP	14.82	6.29	45.39	6.80	6.62	-9.53	7.86	81.10			
VDKT	47.89	42.12	64.27	1.66	49.30	9.62	69.67	38.53	266.39		
AGRAM	0.11	0.11	-0.04	-0.05	0.09	0.17	0.14	-0.05	0.26	0.00	
LOCUSTA	-0.03	-0.05	-0.01	0.05	-0.22	0.69	-0.08	-0.10	-0.46	0.00	0.02

The variance of the rate of return which consists of *n* stocks and shares in cash funds is shown in next formula:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} w_i w_j Cov\,(R_i, R_j) \tag{3}$$

To calculate the optimal portfolio first was necessary to calculate the variance of the portfolio that has the same shares represented in the portfolio. Since first portfolio was calculated with equally represented shares, which means $w_i = w_j$ so that phrase can be calculated as $w_{ij} = w^2$ on the basis of formula (2) calculation of the portfolio variance is as follows (all numbers are rounded to two decimal places):

σ_p^2 = $(0.09^2 \cdot 63.22) + (0.09^2 \cdot 39.69) + (0.09^2 \cdot 183.77) + (0.09^2 \cdot 33.28) + (0.09^2 \cdot 59.71) + (0.09^2 \cdot 127.87)$
$+ (0.09^2 \cdot 148.70) + (0.09^2 \cdot 81.10) + (0.09^2 \cdot 266.39) + (0.09^2 \cdot 0.00) + (0.09^2 \cdot 0.02) + (2 \cdot 0.09^2 \cdot 13.08)$
$+ (2 \cdot 0.09^2 \cdot 15.22) + (2 \cdot 0.09^2 \cdot 1.18) + (2 \cdot 0.09^2 \cdot 32.84) + [2 \cdot 0.09^2 \cdot (-6.81)] + (2 \cdot 0.09^2 \cdot 12.43) +$
$(2 \cdot 0.09^2 \cdot 14.82) + (2 \cdot 0.09^2 \cdot 47.89) + (2 \cdot 0.09^2 \cdot 0.11) + [2 \cdot 0.09^2 \cdot (-0.03)] + (2 \cdot 0.09^2 \cdot 44.35) +$
$[2 \cdot 0.09^2 \cdot (-3.85)] + (2 \cdot 0.09^2 \cdot 26.58) + [2 \cdot 0.09^2 \cdot (-7.02)] + (2 \cdot 0.09^2 \cdot 24.6) + (2 \cdot 0.09^2 \cdot 6.29) +$
$(2 \cdot 0.09^2 \cdot 42.12) + (2 \cdot 0.09^2 \cdot 0.11) + [2 \cdot 0.09^2 \cdot (-0.05)] + (2 \cdot 0.09^2 \cdot 4.72) + (2 \cdot 0.09^2 \cdot 40.81) +$
$(2 \cdot 0.09^2 \cdot 11.82) + (2 \cdot 0.09^2 \cdot 56) + (2 \cdot 0.09^2 \cdot 45.39) + (2 \cdot 0.09^2 \cdot 64.27) + [2 \cdot 0.09^2 \cdot (-0.04)] +$
$[2 \cdot 0.09^2 \cdot (-0.01)] + (2 \cdot 0.09^2 \cdot 2.78) + (2 \cdot 0.09^2 \cdot 4.5) + [2 \cdot 0.09^2 \cdot (-9.69)] + (2 \cdot 0.09^2 \cdot 6.8) +$
$(2 \cdot 0.09^2 \cdot 1.66) + [2 \cdot 0.09^2 \cdot (-0.05)] + (2 \cdot 0.09^2 \cdot 0.05) + [2 \cdot 0.09^2 \cdot (-2.5)] + (2 \cdot 0.09^2 \cdot 24.86) +$
$(2 \cdot 0.09^2 \cdot 6.62) + (2 \cdot 0.09^2 \cdot 49.3) + (2 \cdot 0.09^2 \cdot 0.09) + [2 \cdot 0.09^2 \cdot (-0.22)] + (2 \cdot 0.09^2 \cdot 8.26) +$
$[2 \cdot 0.09^2 \cdot (-9.53)] + (2 \cdot 0.09^2 \cdot 9.62) + (2 \cdot 0.09^2 \cdot 0.17) + (2 \cdot 0.09^2 \cdot 0.69) + (2 \cdot 0.09^2 \cdot 7.86) +$
$(2 \cdot 0.09^2 \cdot 69.67) + (2 \cdot 0.09^2 \cdot 0.14) + [2 \cdot 0.09^2 \cdot (-0.08)] + (2 \cdot 0.09^2 \cdot 38.53) + [2 \cdot 0.09^2 \cdot (-0.05)] +$
$[2 \cdot 0.09^2 \cdot (-0.10)] + (2 \cdot 0.09^2 \cdot 0.26) + [2 \cdot 0.09^2 \cdot (-0.46)] + (2 \cdot 0.09^2 \cdot 0)$

σ^2 = 19.80

σ = 4.45

From this calculation it can be concluded that the portfolio deviation with equal shares of assets is less than the minimum deviation of individual stocks, but not smaller than the yield deviation of cash funds what was expected.

Portfolio rate of return is calculated as a weighted average of expected returns of securities in the portfolio, the weights are shares of securities in the portfolio. In continuation is the formula of the portfolio expected return:

$$E(r_p) = w_i E(r_i) + w_j E(r_j) \dots \dots \dots \dots w_n E(r_n) \quad (4)$$

or generally for *n* securities in the portfolio

$$E(r_p) = \sum_{i=1}^{n} w_i E(R_i) \quad (5)$$

Expected return for portfolio with equal stock proportions:

$E(r_p)$ = (0.09·0.42) + (0.09·1.34) + (0.09·3.56) + (0.09·0.55) + (0.09·0.65) + (0.09·1.10) + (0.09·0.49) + (0.09·1.51) + (0.09·1.46) + (0.09·0.29) + (0.09·0.34)

$E(r_p)$ = 1.06

Based on the aforementioned equation *E(r)* is the expected return, while *w* represents the share. Tags found in subscript – *p* indicates portfolio, *i* and *j* indicates securities. Every security has to be assessed on the basis of contributions of expected return and risk of the entire portfolio.

Calculation showed four portfolio combinations with various asset shares. The first portfolio P_1 had equal shares for 9 stocks and 2 cash funds which was 9.09% share per asset. There were no restrictions or special aims. P_2 had aim to earn the maximum return with a given standard deviation. P_3 had aim to achieve minimum standard deviation with a given level of earnings. P_4 had aim to achieve maximum Sharpe ratio except that aim had no further restrictions.

In all portfolios worth that every stock participates in portfolio under condition:

$$\sum_{i=1}^{n} w_i = 1 \quad (6)$$

Short selling was not allowed, so all shares were positive sizes, as it follows:

$$w_i \geq 0 \quad (7)$$

Table 7. Portfolio optimization

Portfolios	P1 Equal w_i	P2 Max $E(r_p)$	P3 Min σ	P4 Max SR
Constraining Variable	None	at σ <=	at $E(r_p)$=	None
Value of Constraint	N/A	5.77	3.00	N/A
		Portfolio shares		
ERNT	0.09	0.00	0.00	0.00
KOEI	0.09	0.26	0.02	0.00
KORF	0.09	0.28	0.75	0.00
KRAS	0.09	0.09	0.00	0.00
LEDO	0.09	0.00	0.00	0.00
LRH	0.09	0.13	0.11	0.00
MAIS	0.09	0.00	0.00	0.00
RIVP	0.09	0.15	0.11	0.00
VDKT	0.09	0.00	0.00	0.00

Table 7. *(continued)*

Portfolios	P1 Equal w_i	P2 Max $E(r_p)$	P3 Min σ	P4 Max SR
Constraining Variable	None	at σ <=	at $E(r_p)$=	None
Value of Constraint	N/A	5.77	3.00	N/A
		Portfolio shares		
AGRAM	0.09	0.00	0.00	0.57
LOCUSTA	0.09	0.08	0.00	0.43
Σw_i	1.00	1.00	1.00	1.00
$E(r_p)$	1.06	1.81	3.00	0.32
σ_p	4.45	5.77	10.85	0.08
$E(r_p)/\sigma_p$	0.24	0.31	0.28	4.10

Based on table 7, analysis of portfolio P_1 where the shares of asset are equal — expected portfolio return is 1.06 and the standard deviation is 4.45. P_1 was just a starting point in the calculation of the other portfolios.

Portfolio P_2 achieved standard deviation which is equal to the share which has the smallest deviation, which is KRAS. KRAS achieves deviation 5.77 and expected return 0.55, while P_2 has expected return 1.81 at the same level of risk. P_2 didn't aim at the deviation of the cash funds because it was almost insignificant. When the aim would be deviation of the cash fund AGRAM 0.05 then all the shares would be placed in AGRAM cash fund.

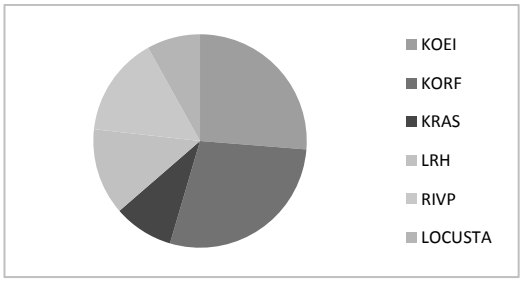

Figure 2. Asset shares in portfolio 2

Portfolio P_3 is a portfolio which has aim to achieve minimum deviation and set minimum return of 3.00. From the offered assets it is not selected coefficient that achieves maximum return (KORF stock 3.56), because in that case all the assets would be allocated to the stock KORF. In order to achieve a high return, level is determined at 3.0, and at this level P_3 achieves a standard deviation of 10.85.

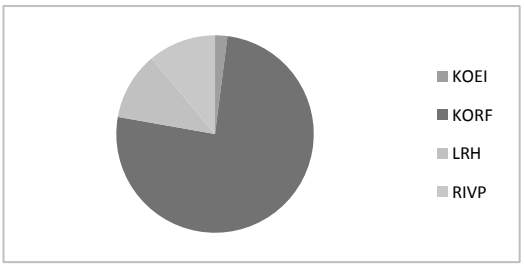

Figure 3. Asset shares in portfolio 3

Portfolio P_4 had the main objective to maximize *Sharpe* ratio, and has made 4.10 — there were no other restrictions. At the maximization of *Sharpe ratio* 0.43% of asset was invested in LOCUSTA and 0.57% of asset in AGRAM. At the maximization of *Sharpe's ratio* portfolio achieves standard deviation of only 0.08, expected return achieves only 0.32.

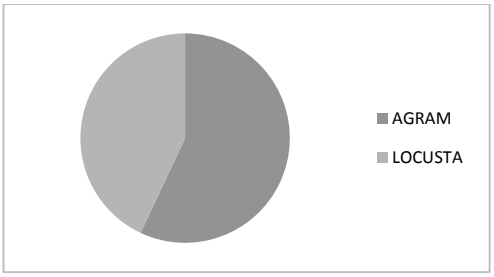

Figure 4. Asset shares in portfolio 4

Of all presented portfolios, portfolio P_2 represents an optimal portfolio and it is taken as the solution of this research. In continuation there are shown two efficient frontiers.

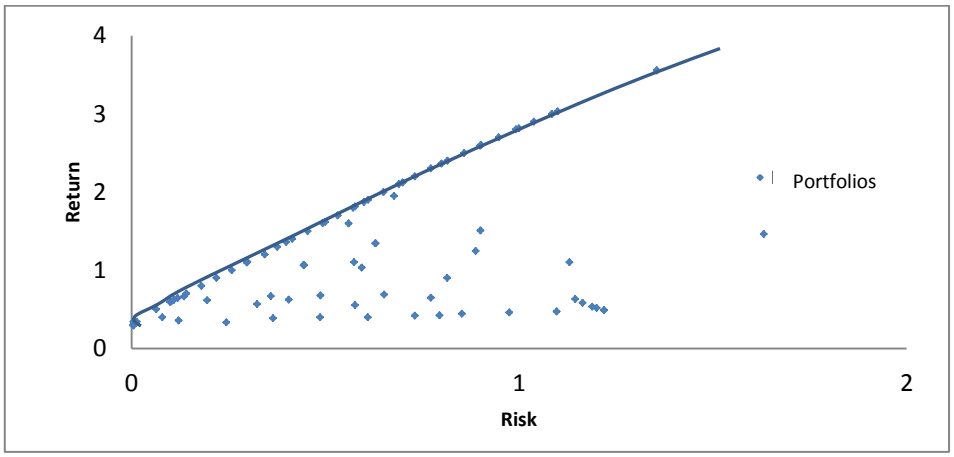

Figure 5. Efficient frontier 1

Figure 5 shows the efficient frontier 1 of portfolio which consist of a combination of risk assets — stocks and risk–free assets — cash funds. Efficient frontier 1 is a part of the research in this paper.

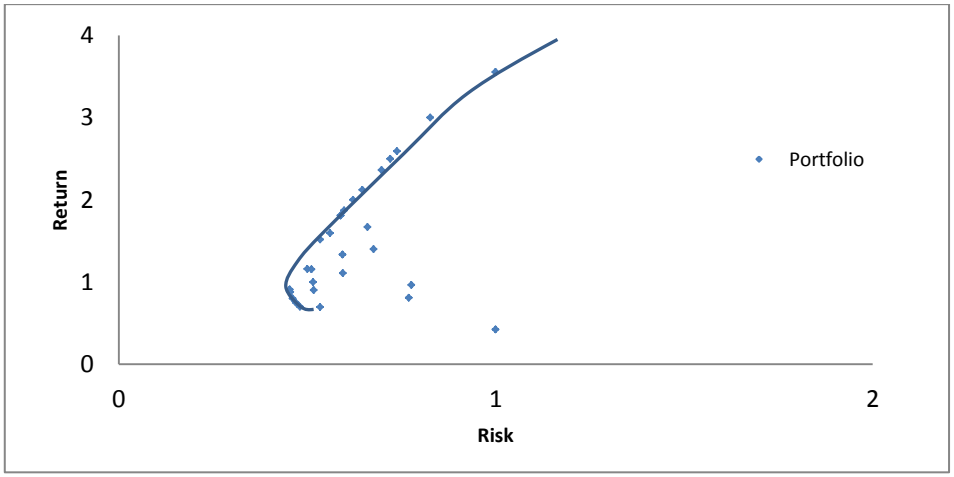

Figure 6. Efficient frontier 2

Figure 6 efficient frontier 2 is the set of portfolios that consist only of risky assets – stocks. The aim of the efficient frontier 2 was to provide an opportunity for comparing with efficient frontier 1. It can be concluded that the efficient frontier 1, which consist of risky and risk–free assets, extending to a zero risk 0.05 because of cash funds. Efficient frontier 2 consists only of stocks, reaches lowest risk at 0.45 with a return of 0.91, and behind this point it is not possible to reduce the risk. If the investor doesn't agree to the minimum risk of efficient frontier 2 he must choose the portfolio at the efficient frontier 1, because it extends to the portfolio which has lowest risk of 0.05, and a return of 0.29. On the efficient frontier 1 we find solution to this research, the optimal portfolio P_2 which offers the best combination which has risk 5.77 and return 1.81.

CONCLUSION

Portfolio management is a very complex task that ranges primarily from defining the length of the time horizon of investment, than defining the amount of risk that investor can handle, choosing the amount of money that would be earmarked for investment, asset allocation etc. The process of asset allocation is not an easy task in which investor in first step decides in what kind of asset wants to invest, and in second step he selects individual type of asset. Investors with a high risk tolerance will choose an asset with higher degree of risk, which also carries a higher return-such as stocks are. More conservative investors will allocate their investment primarily in bonds, cash–funds, and cash equivalents, which have a lower level of risk, and accordingly lower returns.

One of the main aims of this study was to optimize portfolio on the Croatian capital market. Despite the drastic drop in stock market challenge was to find stocks that could have positive returns. Very important role in the forming of optimal portfolio had fundamental financial analysis, rejecting the securities with unfavorable indicators. Using fundamental analysis there were selected securities with the best indicators of growth prospects.

This paper also presented a method of eliminating specific risk, all for the purpose of finding the optimal portfolio. The ultimate aim was finding a portfolio that for a given level of risk provides maximum return. Calculated return was higher than the Croatian market average return. In the selection of the optimal portfolio, two types of asset were included. Cash funds as a form of risk–free investment and stocks as asset with higher profit and risk. Using a risk-free asset an investor has a longer efficient frontier to choose their optimal combination of yield and risk. This work can be used as a good base for future research in the field of modern portfolio theory.

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